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Matrix representation of Osp(2/2) in the U(1/1) basis

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Abstract. Vector coherent state theory is applied to matrix representations of Osp(2/2) in the U(1/1) basis. The branching rule of Osp(2/2)↓U(1/1) is derived. Finite-dimensional irreducible matrix representations of Osp(2/2) in the U(1/1) basis are discussed and matrix elements of Osp(2/2) generators are obtained by using the K-matrix technique.

1. Introduction

Lie superalgebras osp($m/2n$) or osp($m/2n$, R) are useful in the exploitation of supersymmetry in physics. For example, the non-compact supergroups osp($m/2n$, R) have been employed in general superfield theories [1], and osp(4/2, R) is also useful in describing the nuclear spectra of the nickel isotopes [2].

Recently, some detailed investigations have been carried out for finite irreducible representations of gl(m/n) in the gl(m)⊕gl(n) basis [3–5], and osp($m/2n$) in the so(m)⊕sp($2n$) basis [6, 7]. In [3] and [7] Le Blanc and Rowe demonstrate that vector coherent state (VCS) theory applies without substantial modification to the representation theory of classical Lie superalgebras in the Lie algebra basis. Recent development [8] shows that VCS theory applies equally well to Lie superalgebras in the superalgebra basis. In this circumstance, no grade star representation exists.

In this paper, we will consider osp(2/2) in the u(1/1) basis, which is the starting point for studying the general case osp($2m/2n$) in the u(m/n) basis. In section 2, we briefly review the main properties of the osp(2/2) and u(1/1) finite-dimensional irreducible representations. In section 3, the irreducible representation of osp(2/2) in the u(1/1) basis will be constructed via VCS theory. In section 4, we will derive matrix elements of osp(2/2) generators.

2. The superalgebras osp(2/2) and u(1/1)

The superalgebra osp(2/2) can be defined by the following commutation and anti-commutation relations:

$$\begin{aligned} &[E_0^1, E_\pm] = \pm E_\pm &&[E_0^2, E_\pm] = \mp E_\pm &&[E_-, E_+]_+ = E_0^1 + E_0^2 \\ &[E_0^1, A_\pm] = \pm A_\pm &&[E_0^2, A_\pm] = \pm A_\pm &&[A_-, A_+]_+ = -E_0^1 + E_0^2 \\ &[B_+, B_-] = 2E_0^1 &&[E_0^1, B_\pm] = \pm 2B_\pm &&[E_0^2, B_\pm] = 0 \end{aligned} \qquad (2.1)$$

$$[A_\pm, E_\pm]_+ = \sqrt{2}\, B_\pm \qquad [B_\pm, E_\mp] = \mp\sqrt{2}\, A_\pm \qquad [B_\pm, A_\mp] = \pm\sqrt{2}\, E_\pm$$

$$[A_\mp, E_\pm]_+ = [B_\pm, E_\pm] = [B_\pm, A_\pm] = 0$$

where

$$\begin{pmatrix} E_+ \\ E_- \end{pmatrix} = \begin{pmatrix} \sqrt{2}\, W_+ \\ -\sqrt{2}\, V_- \end{pmatrix} \qquad \begin{pmatrix} A_+ \\ A_- \end{pmatrix} = \begin{pmatrix} \sqrt{2}\, V_+ \\ \sqrt{2}\, W_- \end{pmatrix}$$
$$\begin{pmatrix} B_+ \\ B_- \end{pmatrix} = \begin{pmatrix} \sqrt{2}\, Q_+ \\ \sqrt{2}\, Q_- \end{pmatrix} \qquad \begin{pmatrix} E_0^1 \\ E_0^2 \end{pmatrix} = \begin{pmatrix} 2Q_0 \\ 2B \end{pmatrix}. \tag{2.2}$$

The symbols $W_\pm$, $V_\pm$, Q_i ($i = 0, -, +$), and B were used in [9] to denote the generators of the osp(2/2).

The quadrupole Casimir operator of osp(2/) can be written as

$$C_2(\text{osp}(2/2)) = \tfrac{1}{4} C_2(\text{u}(1/1)) + \tfrac{1}{4}(A_+A_- - A_-A_+) + \tfrac{1}{4}(B_-B_+ + B_+B_-) \tag{2.3}$$

where

$$C_2(\text{u}(1/1)) = (E_0^1)^2 + E_-E_+ - E_+E_- - (E_0^2)^2 \tag{2.4}$$

is the quadrupole Casimir operator of u(1/1).

There are two classes of star representations of osp(2/2), which are composed of the (b, j) representations for which b is real and $\pm b \geqslant j$ [9].

From (2.1) we can see that $\{E_0^i\ (i = 1, 2), E_\pm\}$ forms u(1/1) subalgebra. The u(1/1) irreducible representation is two dimensional. The basis vectors can be expressed as

$$\left|\begin{matrix} (m_1|m_2) \\ m_1 \end{matrix}\right\rangle \qquad \text{and} \qquad \left|\begin{matrix} (m_1|m_2) \\ m_1 - 1 \end{matrix}\right\rangle.$$

We also have [8]

$$E_+\left|\begin{matrix} (m_1|m_2) \\ m_1 - 1 \end{matrix}\right\rangle = (S(m_1+m_2)(m_1+m_2))^{1/2}\left|\begin{matrix} (m_1|m_2) \\ m_1 \end{matrix}\right\rangle \tag{2.5a}$$

$$E_-\left|\begin{matrix} (m_1|m_2) \\ m_1 \end{matrix}\right\rangle = S(m_1+m_2)(S(m_1+m_2)(m_1+m_2))^{1/2}\left|\begin{matrix} (m_1|m_2) \\ m_1 - 1 \end{matrix}\right\rangle \tag{2.5b}$$

where $S(x)$ is the sign of x, namely

$$S(x) = \begin{cases} 1 & \text{for } x \geqslant 0 \\ -1 & \text{for } x < 0. \end{cases} \tag{2.6}$$

There are also two classes of star representations for u(1/1) corresponding to $m_1 + m_2 \geqslant 0$ and $m_1 + m_2 < 0$, respectively. For u(1/1) all grade star representations are also star representations and vice versa.

Let the lowest-weight state vectors for osp(2/2) be

$$|\text{lw}\rangle = \left|\begin{matrix} (b, j) \\ (-2j+1+\alpha|2b-1+\alpha) \\ m \end{matrix}\right\rangle \equiv \left|\begin{matrix} (-2j+1+\alpha|2b-1+\alpha) \\ m \end{matrix}\right\rangle \tag{2.7a}$$

where $\alpha = 0$, and $m = -2j+1+\alpha$ or $-2j+\alpha$; and let the highest-weight state vectors for osp(2/2) be

$$|\mathrm{hw}\rangle = \left|\begin{matrix}(b,j)\\(2j|2b)\\m'\end{matrix}\right\rangle \equiv \left|\begin{matrix}(2j|2b)\\m\end{matrix}\right\rangle \tag{2.7b}$$

where $m' = 2j+2b$ or $2j+2b-1$. Obviously, $|\mathrm{lw}\rangle$ and $|\mathrm{hw}\rangle$ satisfy

$$\begin{pmatrix}A_-\\B_-\end{pmatrix}|\mathrm{lw}\rangle = 0 \tag{2.8a}$$

and

$$\begin{pmatrix}A_+\\B_+\end{pmatrix}|\mathrm{hw}\rangle = 0 \tag{2.8b}$$

respectively.

It should be noted that the complete reducibility into irreps of u(1/1) will only take place for irreps of osp(2/2) which are stars for the u(1/1) subalgebra; i.e. when $b \geqslant j$ or $j+b<0$. It can easily be verified that

$$C_2(\mathrm{osp}(2/2))\begin{pmatrix}|\mathrm{lw}\rangle\\|\mathrm{hw}\rangle\end{pmatrix} = (j^2-b^2)\begin{pmatrix}|\mathrm{lw}\rangle\\|\mathrm{hw}\rangle\end{pmatrix}. \tag{2.9}$$

3. VCS representations of osp(2/2) in u(1/1) basis

In VCS theory the Z-gradation of the Lie algebra or Lie superalgebra g is

$$g = n_0 + n_+ + n_- \tag{3.1}$$

where n_0, containing Cartan subalgebra of g, is the stability subalgebra of g, and $n_\pm$ are nilpotent subalgebras of raising and lowering type operators, respectively. However, when both g and n_0 are Lie superalgebras, the Z-gradation of g defined by (3.1) is not consistent with the Z_2-gradation of the algebra. When $g = \mathrm{osp}(2/2)$, and $n_0 = \mathrm{u}(1/1)$, the Z-grading operator can be defined as

$$\hat{\mathscr{Z}} = \tfrac{1}{2}(E_0^1 + E_0^2) \tag{3.2}$$

while the Z_2-grading operator is $\hat{Z} = E_0^1$ (see [7]).

The novel features of VCS theory applied to Lie superalgebras in superalgebra basis are the following. Firstly, the coset representative $\exp X$ of G/N_0, with $X \in n_+$ (or n_-), is parametrized by both Bargmann and Grassmann variables since n_+ (or n_-) contains elements from both odd and even parts of the superalgebra g. Secondly, in contrast to the VCS theory applied to Lie superalgebras in the Lie algebra basis, in this case the Bargmann variables and Grassmann variables transform as the components of the same irreducible tensor of n_0.

In the following, we will construct VCS representations of osp(2/2) in the u(1/1) basis.

According to the VCS theory, the VCS wavefunction can be built on the lowest-weight state vectors

$$\Psi(z,\theta) = \sum_m \left\langle\begin{matrix}(-2j+1|2b-1)\\m\end{matrix}\right| \mathrm{e}^{zB_-+\theta A_-}|\Psi\rangle \left|\begin{matrix}(-2j+1|2b-1)\\m\end{matrix}\right\rangle \tag{3.3}$$

where z and θ are Bargmann and Grassmann variables, respectively. Using the commutation and anticommutation relations given by (2.1), we readily obtain the following VCS representations of osp(2/2):

$$\begin{aligned}
&\Gamma(E_0^1)=\mathscr{E}_0^1+2z\partial_z+\theta\partial_\theta\\
&\Gamma(E_0^2)=\mathscr{E}_0^2+\theta\partial_\theta\\
&\Gamma(E_+)=\mathscr{E}_++\sqrt{2}\,z\partial_\theta\\
&\Gamma(E_-)=\mathscr{E}_-+\sqrt{2}\,\theta\partial_z\\
&\Gamma(A_-)=\partial_\theta \qquad \Gamma(B_-)=\partial_z\\
&\Gamma(A_+)=-\sqrt{2}\,z\mathscr{E}_-+\theta(\mathscr{E}_0^2-\mathscr{E}_0^1)-2\theta z\partial_z\\
&\Gamma(B_+)=-\sqrt{2}\,\theta\mathscr{E}_+-2z\mathscr{E}_0^1-2z\theta\partial_\theta-2z^2\partial_z
\end{aligned} \tag{3.4}$$

where $\{\mathscr{E}_\pm,\mathscr{E}_0^i\ (i=1,2)\}$ span an intrinsic algebra u(1/1) only acting on the lowest-weight states defined by (2.6).

As shown in [8], the indefinite metric should be introduced in order to derive u(1/1) CG coefficients. The relation between the metrics of two subspaces is

$$\mathscr{E}(m_1m_2,m_1)\mathscr{E}(m_1m_2,m_1-1)=S(m_1+m_2) \tag{3.5}$$

where

$$\mathscr{E}(m_1m_2,m_1-\alpha)=\left\langle\begin{matrix}(m_1|m_2)\\ m_1-\alpha\end{matrix}\middle|\begin{matrix}(m_1|m_2)\\ m_1-\alpha\end{matrix}\right\rangle \tag{3.6}$$

with $\alpha=0$ or 1. The matrix element of an operator T should be written as

$$\mathscr{E}(\Lambda'q')\left\langle\begin{matrix}\Lambda'\\ q'\end{matrix}\middle|T\middle|\begin{matrix}\Lambda\\ q\end{matrix}\right\rangle. \tag{3.7}$$

We assume that T always acts on the right vector $\left|\begin{matrix}\Lambda\\ q\end{matrix}\right\rangle$.

The definition of the irreducible tensor is

$$[E,T_q^\Lambda]=\mathscr{E}(\Lambda q')\left\langle\begin{matrix}\Lambda\\ q'\end{matrix}\middle|E\middle|\begin{matrix}\Lambda\\ q\end{matrix}\right\rangle T_{q'}^\Lambda \tag{3.8}$$

where

$$[E,T_q^\Lambda]=ET_q^\Lambda-(-)^{j\sigma(\Lambda q)}T_q^\Lambda E. \tag{3.9}$$

E is u(1/1) generator, and $\sigma(\Lambda q)$ is the grade of T_q^Λ.

From (3.8) we know that $(z^n/\sqrt{n!},\ z^{n-1}\theta/\sqrt{(n-1)!})$ and $(\partial_\theta,\partial_z)$ are u(1/1) $(2n|0)$ and $(-1|-1)$ tensors corresponding to type $1(m_1+m_2\geqslant 0)$ and type 2 $(m_1+m_2<0)$ star representations, respectively. Using (3.5), we successively obtain

$$\left|\begin{matrix}(m_1+2n|m_2)\\ m_1+2n\end{matrix}\right\rangle=\frac{z^n}{\sqrt{n!}}\left|\begin{matrix}(m_1|m_2)\\ m_1\end{matrix}\right\rangle$$

$$\left|\begin{matrix}(m_1+2n-1|m_2+1)\\ m_1+2n-2\end{matrix}\right\rangle=\frac{z^{n-1}\theta}{\sqrt{(n-1)!}}\left|\begin{matrix}(m_1|m_2)\\ m_1-1\end{matrix}\right\rangle$$

$$\left|\begin{matrix}(m_1+2n|m_2)\\ m_1+2n-1\end{matrix}\right\rangle=\left[\frac{m_1+m_2}{m_1+m_2+2n}\right]^{1/2}\frac{z^n}{\sqrt{n!}}\left|\begin{matrix}(m_1|m_2)\\ m_1-1\end{matrix}\right\rangle$$
$$+\left[\frac{2n}{m_1+m_2+2n}\right]^{1/2}\frac{z^{n-1}\theta}{\sqrt{(n-1)!}}\left|\begin{matrix}(m_1|m_2)\\ m_1\end{matrix}\right\rangle \qquad (3.10)$$

$$\left|\begin{matrix}(m_1+2n-1|m_2+1)\\ m_1+2n-1\end{matrix}\right\rangle$$
$$=-\left[\frac{m_1+m_2}{m_1+m_2+2n}\right]^{1/2}\frac{z^{n-1}\theta}{\sqrt{(n-1)!}}\left|\begin{matrix}(m_1|m_2)\\ m_1\end{matrix}\right\rangle$$
$$+\left[\frac{2n}{m_1+m_2+2n}\right]^{1/2}\frac{z^n}{\sqrt{n!}}\left|\begin{matrix}(m_1|m_2)\\ m_1-1\end{matrix}\right\rangle.$$

Equation (3.11) also gives orthonormal BG basis vectors for osp(2/2) when $(m_1|m_2)=(-2j+1|2b-1)$ and $b\geqslant j$.

The u(1/1) CG coefficients satisfy the following orthogonality conditions

$$\sum_{M_1M_2}\mathscr{E}(\Lambda_1M_1)\mathscr{E}(\Lambda_2M_2)\left\langle\begin{matrix}\Lambda_1 & \Lambda_2\\ M_1 & M_2\end{matrix}\middle|\begin{matrix}\Lambda\\ M\end{matrix}\right\rangle\left\langle\begin{matrix}\Lambda_1 & \Lambda_2\\ M_1 & M_2\end{matrix}\middle|\begin{matrix}\Lambda'\\ M'\end{matrix}\right\rangle=\mathscr{E}(\Lambda M)\delta_{\Lambda\Lambda'}\delta_{MM'} \qquad (3.11a)$$

$$\sum_{\Lambda}\mathscr{E}(\Lambda M)\left\langle\begin{matrix}\Lambda_1 & \Lambda_2\\ M_1 & M_2\end{matrix}\middle|\begin{matrix}\Lambda\\ M\end{matrix}\right\rangle\left\langle\begin{matrix}\Lambda_1 & \Lambda_2\\ M_1' & M_2'\end{matrix}\middle|\begin{matrix}\Lambda\\ M\end{matrix}\right\rangle=\mathscr{E}(\Lambda_1M_1)\mathscr{E}(\Lambda_2M_2)\delta_{M_1M_1'}\delta_{M_2M_2'}. \qquad (3.11b)$$

The u(1/1) reduced matric elements are defined by the following Wigner-Eckart theorem [8],

$$\mathscr{E}(\Lambda M)\left\langle\begin{matrix}\Lambda\\ M\end{matrix}\middle|T^{\Lambda_2}_{M_2}\middle|\begin{matrix}\Lambda_1\\ M_1\end{matrix}\right\rangle$$
$$=\langle\Lambda\|T^{\Lambda_2}\|\Lambda_1\rangle\mathscr{E}(\Lambda M)\mathscr{E}(\Lambda_1M_1)\mathscr{E}(\Lambda_2M_2)\left\langle\begin{matrix}\Lambda_1 & \Lambda_2\\ M_1 & M_2\end{matrix}\middle|\begin{matrix}\Lambda\\ M\end{matrix}\right\rangle. \qquad (3.12)$$

It should be stressed that the u(1/1) CG coefficients and Wigner-Eckart theorem given by (3.12) applies to star representations of one type only (for $b\geqslant j$ or $j+b<0$); the decomposition of a type 1 star irrep ($b\geqslant j$) with a type 2 star irrep ($b+j<0$) is not completely reducible. Thus orthonormal BG vectors corresponding to type 2 star irreps ($b+j<0$) cannot be obtained from (3.10). But we can construct them in the highest-weight space of osp(2/2). In this case the VCS wavefunctions can be written as

$$\Psi(z,\theta)=\sum_m\left\langle\begin{matrix}(2j|2b)\\ m\end{matrix}\middle|e^{zB_++\theta A_+}|\Psi\rangle\middle|\begin{matrix}(2j|2b)\\ m\end{matrix}\right\rangle. \qquad (3.13)$$

Using the commutation and anticommutation relations given by (2.1), we similarly obtain the following VCS representations of osp(2/2):

$$\begin{aligned}
&\Gamma(E_0^1)=\mathscr{E}_0^1-2z\partial_z-\theta\partial_\theta\\
&\Gamma(E_0^2)=\mathscr{E}_0^2-\theta\partial_\theta\\
&\Gamma(E_+)=\mathscr{E}_++\sqrt{2}\,\theta\partial_z\\
&\Gamma(E_-)=\mathscr{E}_--\sqrt{2}\,z\partial_\theta\\
&\Gamma(A_+)=\partial_\theta \qquad \Gamma(B_+)=\partial_z\\
&\Gamma(A_-)=\sqrt{2}\,z\mathscr{E}_++\theta(\mathscr{E}_0^2-\mathscr{E}_0^1)+2\theta z\partial_z\\
&\Gamma(B_-)=\sqrt{2}\,\theta\mathscr{E}_-+2z\mathscr{E}_0^1-2z^2\partial_z-2z\theta\partial_\theta.
\end{aligned} \qquad (3.14)$$

From (3.8) we know that $(z^n/\sqrt{n!},\, z^{n-1}\theta/\sqrt{(n-1)!})$ is the u(1/1) $(-2n+1|-1)$ tensor corresponding to type 2 $(m_1+m_2<0)$ star representations. Similar to (3.10) we obtain

$$\left|\begin{matrix}(m_1-2n|m_2)\\ m_1-2n-1\end{matrix}\right\rangle=\frac{z^n}{\sqrt{n!}}\left|\begin{matrix}(m_1|m_2)\\ m_1-1\end{matrix}\right\rangle$$

$$\left|\begin{matrix}(m_1-2n|m_2)\\ m_1-2n\end{matrix}\right\rangle$$

$$=\left[\frac{2n}{S(m_1+m_2-2n)(m_1+m_2-2n)}\right]^{1/2}\frac{z^{n-1}\theta}{\sqrt{(n-1)!}}\left|\begin{matrix}(m_1|m_2)\\ m_1-1\end{matrix}\right\rangle$$

$$+\left[\frac{S(m_1+m_2)(m_1+m_2)}{S(m_1+m_2-2n)(m_1+m_2-2n)}\right]^{1/2}\frac{z^n}{\sqrt{n!}}\left|\begin{matrix}(m_1|m_2)\\ m_1\end{matrix}\right\rangle \tag{3.15}$$

$$\left|\begin{matrix}(m_1-2n+1|m_2-1)\\ m_1-2n+1\end{matrix}\right\rangle=\frac{z^{n-1}\theta}{\sqrt{(n-1)!}}\left|\begin{matrix}(m_1|m_2)\\ m_1\end{matrix}\right\rangle$$

$$\left|\begin{matrix}(m_1-2n+1|m_2-1)\\ m_1-2n\end{matrix}\right\rangle$$

$$=-\left[\frac{S(m_1+m_2)(m_1+m_2)}{S(m_1+m_2-2n)(m_1+m_2-2n)}\right]^{1/2}\frac{z^{n-1}\theta}{\sqrt{(n-1)!}}\left|\begin{matrix}(m_1|m_2)\\ m_1-1\end{matrix}\right\rangle$$

$$+\left[\frac{2n}{S(m_1+m_2-2n)(m_1+m_2-2n)}\right]^{1/2}\frac{z^n}{\sqrt{n!}}\left|\begin{matrix}(m_1|m_2)\\ m_1\end{matrix}\right\rangle.$$

Equation (3.15) also gives the orthonormal BG vectors for osp(2/2) when $(m_1|m_2)=(2j|2b)$ and $b+j<0$.

4. Matrix representations of osp(2/2) in the u(1/1) basis

In order to obtain the Hermitian representation $\gamma=K^{-1}\Gamma K$, we need to derive K^2 matrices which can be obtained from the following equation [3],

$$\langle x|K^2\Gamma^\dagger(G_-)|y\rangle=\varepsilon\langle x|\Gamma(G_+)K^2|y\rangle \tag{4.1}$$

where $G_\pm=A_\pm$ or $B_\pm$, and ε takes a constant value +1 or −1 for all matrix elements for a star representation. As pointed out in [8], no grade star representation exists which is not already a star in this case. In the following, we discuss the condition for star representations.

For type 1 star representations $(b\geqslant j)$ the recursion relations for K^2 matrices are

$$\frac{K^2(2n-2j+2,2b)}{K^2(2n-2j+1,2b-1)}=2(\pm)(b+j) \tag{4.2a}$$

$$\frac{K^2(2(n+1)-2j+\alpha+1,2b+\alpha-1)}{K^2(2n-2j+\alpha+1,2b+\alpha-1)}=2(\pm)(2j-n-1) \tag{4.2b}$$

for $\alpha=0$ or 1. Because $j>0$ and $b\geqslant j$, we should choose positive sign on the RHS of (4.2a, b), which corresponds to the condition that

$$(\gamma(G_\pm))^\dagger=\gamma(G_\mp). \tag{4.3}$$

From (4.2) we obtain

$$K^2(2n-2j+\alpha+1, 2b+\alpha-1) = \begin{cases} 2^n(2j-1)!/(2j-n-1)! & \text{for } \alpha=0 \\ 2^n[(2j-1)!/(2j-n-1)!]2(b+j) & \text{for } \alpha=1. \end{cases} \quad (4.4)$$

For type 2 star representations ($b+j<0$) the recursion relations for K^2 matrices are

$$\frac{K^2(2j-2(n+1)-\alpha, 2b-\alpha)}{K^2(2j-2n-\alpha, 2b-\alpha)} = -2(\pm)(2j-n-1) \quad (4.5a)$$

$$\frac{K^2(2j-2n-1, 2b-1)}{K^2(2j-2n, 2b)} = 2(\pm)(b-j) \quad (4.5b)$$

for $\alpha=0$ or 1. Because $j>0$ and $b<-j$, we should choose a minus sign on the RHS of (4.5*a*, *b*), which corresponds to the condition that

$$(\gamma(G_\pm))^\dagger = -\gamma(G_\mp). \quad (4.6)$$

Equation (4.5) gives

$$K^2(2j-2n-\alpha, 2b-\alpha) = \begin{cases} 2^n(2j-1)!/(2j-n-1)! & \text{for } \alpha=0 \\ 2^n[(2j-1)!/(2j-n-1)!]2(j-b) & \text{for } \alpha=1. \end{cases} \quad (4.7)$$

Combining (3.10), (4.4), (3.15) and (4.7), we obtain the following branching rule for osp(2/2)↓u(1/1)

$$\text{osp}(2/2)\downarrow\text{u}(1/1)$$

$$(b,j) = \sum_{n=0}^{2j-1} [(-2j+1+2n|2b-1)+(-2j+2n+2|2b)] \quad (4.8a)$$

for both $b \geqslant j$ and $b+j<0$.

In the following, we give all the non-zero matrix elements of z and θ for type 1 and 2 star representations, respectively. For type 1 star representations we can use the positive definite metric since $m_1+m_2 \geqslant 0$ is always satisfied. In this case we will write the matrix elements of T as $\langle m'|T|m\rangle$ instead of $\mathscr{E}(m')\langle m'|T|m\rangle$. They are

$$\left\langle \begin{matrix}(m_1+2n+2|m_2)\\ m_1+2n+2\end{matrix} \middle| z \middle| \begin{matrix}(m_1+2n|m_2)\\ m_1+2n\end{matrix} \right\rangle = (n+1)^{1/2}$$

$$\left\langle \begin{matrix}(m_1+2n+1|m_2+1)\\ m_1+2n\end{matrix} \middle| z \middle| \begin{matrix}(m_1+2n-1|m_2+1)\\ m_1+2n-2\end{matrix} \right\rangle = (n)^{1/2}$$

$$\left\langle \begin{matrix}(m_1+2n+2|m_2)\\ m_1+2n+1\end{matrix} \middle| z \middle| \begin{matrix}(m_1+2n|m_2)\\ m_1+2n-1\end{matrix} \right\rangle = \left[\frac{(m_1+m_2+2n)(n+1)}{(m_1+m_2+2n+2)}\right]^{1/2}$$

$$\left\langle \begin{matrix}(m_1+2n+1|m_2+1)\\ m_1+2n+1\end{matrix} \middle| z \middle| \begin{matrix}(m_1+2n-1|m_2+1)\\ m_1+2n-1\end{matrix} \right\rangle = \left[\frac{n(m_1+m_2+2n+2)}{(m_1+m_2+2n)}\right]^{1/2}$$

$$\left\langle \begin{matrix}(m_1+2n+1|m_2+1)\\ m_1+2n+1\end{matrix} \middle| z \middle| \begin{matrix}(m_1+2n|m_2)\\ m_1+2n-1\end{matrix} \right\rangle = \left[\frac{2(m_1+m_2)}{(m_1+m_2+2n)(m_1+m_2+2n+2)}\right]^{1/2}$$

(4.8*b*)

$$\left\langle \begin{matrix}(m_1+2n+2|m_2)\\ m_1+2n+1\end{matrix} \middle| \theta \middle| \begin{matrix}(m_1+2n|m_2)\\ m_1+2n\end{matrix} \right\rangle = \left[\frac{2(n+1)}{m_1+m_2+2n+2}\right]^{1/2}$$

$$\left\langle \begin{matrix}(m_1+2n+1|m_2+1)\\ m_1+2n+1\end{matrix} \middle| \theta \middle| \begin{matrix}(m_1+2n|m_2)\\ m_1+2n\end{matrix} \right\rangle = -\left[\frac{m_1+m_2}{m_1+m_2+2n+2}\right]^{1/2}$$

$$\left\langle \begin{matrix}(m_1+2n+1|m_2+1)\\ m_1+2n\end{matrix} \middle| \theta \middle| \begin{matrix}(m_1+2n|m_2)\\ m_1+2n-1\end{matrix} \right\rangle = \left[\frac{m_1+m_2}{m_1+m_2+2n}\right]^{1/2}$$

$$\left\langle \begin{matrix}(m_1+2n+1|m_2+1)\\ m_1+2n\end{matrix} \middle| \theta \middle| \begin{matrix}(m_1+2n-1|m_2+1)\\ m_1+2n-1\end{matrix} \right\rangle = \left[\frac{2n}{m_1+m_2+2n}\right]^{1/2}.$$

For type 2 star representations $(b+j<0)$ we have

$$\mathscr{E}(m_1-2n-2, m_2; m_1-2n-3)\left\langle \begin{matrix}(m_1-2n-2|m_2)\\ m_1-2n-3\end{matrix} \middle| z \middle| \begin{matrix}(m_1-2n|m_2)\\ m_1-2n-1\end{matrix} \right\rangle = (n+1)^{1/2}$$

$$\mathscr{E}(m_1-2n-2, m_2; m_1-2n-2)\left\langle \begin{matrix}(m_1-2n-2|m_2)\\ m_1-2n-2\end{matrix} \middle| z \middle| \begin{matrix}(m_1-2n|m_2)\\ m_1-2n\end{matrix} \right\rangle$$

$$= \left[\frac{(n+1)(2n-m_1-m_2)}{2n+2-m_1-m_2}\right]^{1/2}$$

$$\mathscr{E}(m_1-2n-1, m_2-1; m_1-2n-2)\left\langle \begin{matrix}(m_1-2n-1|m_2-1)\\ m_1-2n-2\end{matrix} \middle| z \middle| \begin{matrix}(m_1-2n+1|m_2-1)\\ m_1-2n\end{matrix} \right\rangle$$

$$= \left[\frac{n(2n+2-m_1-m_2)}{2n-m_1-m_2}\right]^{1/2}$$

$$\mathscr{E}(m_1-2n-1, m_2-1; m_1-2n-1)\left\langle \begin{matrix}(m_1-2n-1|m_2-1)\\ m_1-2n-1\end{matrix} \middle| z \middle| \begin{matrix}(m_1-2n+1|m_2-1)\\ m_1-2n+1\end{matrix} \right\rangle$$

$$= (n)^{1/2}$$

$$\mathscr{E}(m_1-2n-1, m_2-1; m_1-2n-2)\left\langle \begin{matrix}(m_1-2n-1|m_2-1)\\ m_1-2n-2\end{matrix} \middle| z \middle| \begin{matrix}(m_1-2n|m_2)\\ m_1-2n\end{matrix} \right\rangle$$

$$= \left[\frac{2(m_1+m_2)}{(2n+2-m_1-m_2)(m_1+m_2-2n)}\right]^{1/2} \tag{4.9}$$

$$\mathscr{E}(m_1-2n-2, m_2; m_1-2n-2)\left\langle \begin{matrix}(m_1-2n-2|m_2)\\ m_1-2n-2\end{matrix} \middle| \theta \middle| \begin{matrix}(m_1-2n|m_2)\\ m_1-2n-1\end{matrix} \right\rangle$$

$$= \left[\frac{2(n+1)}{2n-m_1-m_2+2}\right]^{1/2}$$

$$\mathscr{E}(m_1-2n-1, m_2-1; m_1-2n-1)\left\langle \begin{matrix}(m_1-2n-1|m_2-1)\\ m_1-2n-1\end{matrix} \middle| \theta \middle| \begin{matrix}(m_1-2n|m_2)\\ m_1-2n\end{matrix} \right\rangle$$

$$= \left[\frac{m_1+m_2}{m_1+m_2-2n}\right]^{1/2}$$

$$\mathscr{E}(m_1-2n-1, m_2-1; m_1-2n-1)\left\langle \begin{matrix}(m_1-2n-1|m_2-1)\\ m_1-2n-1\end{matrix} \middle| \theta \middle| \begin{matrix}(m_1-2n+1|m_2-1)\\ m_1-2n\end{matrix} \right\rangle$$

$$= \left[\frac{2n}{2n-m_1-m_2}\right]^{1/2}$$

$$\mathscr{E}(m_1-2n-1, m_2-1; m_1-2n-2)\left\langle \begin{matrix}(m_1-2n-1|m_2-1)\\ m_1-2n-2\end{matrix}\right|\theta\left|\begin{matrix}(m_1-2n|m_2)\\ m_1-2n-1\end{matrix}\right\rangle$$

$$=-\left[\frac{m_1+m_2}{m_1+m_2-2n-2}\right]^{1/2}.$$

The reduced matric elements of $T^{(2|0)}=(z,\theta)$ in the type 1 star representation are

$$\langle (m_1+2n+2|m_2)\| T^{(2|0)}\|(m_1+2n|m_2)\rangle=(n+1)^{1/2}$$

$$\langle (m_1+2n+1|m_2+1)\| T^{(2|0)}\|(m_1+2n-1|m_2+1)\rangle$$

$$=\left[\frac{n(m_1+m_2+2n+2)}{m_1+m_2+2n}\right]^{1/2} \tag{4.10}$$

$$\langle (m_1+2n+1|m_2+1)\| T^{(2|0)}\|(m_1+2n|m_2)\rangle=\left[\frac{m_1+m_2}{m_1+m_2+2n}\right]^{1/2}.$$

While the reduced matrix elements of $T^{(-1|-1)}=(z,\theta)$ in the type 2 star representations are

$$\langle (m_1-2n-2|m_2)\| T^{(-1|-1)}\|(m_1-2n|m_2)\rangle=(n+1)^{1/2}$$

$$\langle (m_1-2n-1|m_2-1)\| T^{(-1|-1)}\|(m_1-2n+1|m_2-1)\rangle$$

$$=\left[\frac{n(2n+2-m_1-m_2)}{2n-m_1-m_2}\right]^{1/2} \tag{4.11}$$

$$\langle (m_1-2n-1|m_2-1)\| T^{(-1|-1)}\|(m_1-2n|m_2)\rangle=\left[\frac{m_1+m_2}{m_1+m_2-2n}\right]^{1/2}.$$

In (4.8) and (4.10) $(m_1|m_2)=(-2j+1|2b-1)$ and $b\geqslant j$, while in (4.9) and (4.11) $(m_1|m_2)=(2j|2b)$ and $b+j<0$.

The matrix elements of $\gamma(A_\pm)$ and $\gamma(B_\pm)$ can then be obtained through the following relations

$$\left\langle \begin{matrix}(m_1'|m_2')\\ m'\end{matrix}\right|\gamma(A_\mp)\left|\begin{matrix}(m_1|m_2)\\ m\end{matrix}\right\rangle$$

$$=\mp\left\langle \begin{matrix}(m_1|m_2)\\ m\end{matrix}\right|\gamma(A_\pm)\left|\begin{matrix}(m_1'|m_2')\\ m'\end{matrix}\right\rangle$$

$$=\frac{K(m_1'm_2')}{K(m_1m_2)}\left\langle \begin{matrix}(m_1'|m_2')\\ m'\end{matrix}\right|\theta\left|\begin{matrix}(m_1|m_2)\\ m\end{matrix}\right\rangle \tag{4.12a}$$

$$\left\langle \begin{matrix}(m_1'|m_2')\\ m'\end{matrix}\right|\gamma(B_\mp)\left|\begin{matrix}(m_1|m_2)\\ m\end{matrix}\right\rangle$$

$$=\mp\left\langle \begin{matrix}(m_1|m_2)\\ m\end{matrix}\right|\gamma(B_\pm)\left|\begin{matrix}(m_1'|m_2')\\ m'\end{matrix}\right\rangle$$

$$=\frac{K(m_1'm_2'}{K(m_1m_2)}\left\langle \begin{matrix}(m_1'|m_2')\\ m'\end{matrix}\right|z\left|\begin{matrix}(m_1|m_2)\\ m\end{matrix}\right\rangle \tag{4.12b}$$

for type 1 (the lower sign) and type 2 (the upper sign) star irreps, respectively. The representation is atypical when $b=j$ for type 1 and $b=-j$ for type 2 star irreps.

We can also obtain the following reduced matrix elements of $\gamma(G_\pm)$, where $G_\pm = A_\pm$ or $B_\pm$:

$$\langle (m_1'|m_2') \| \gamma(G_+) \| (m_1|m_2) \rangle = \frac{K(m_1' m_2')}{K(m_1 m_2)} \langle (m_1'|m_2' \| T^{(2|0)} \| (m_1|m_2) \rangle \tag{4.13}$$

for type 1 star irreps, and

$$\langle (m_1'|m_2') \| \gamma(G_-) \| (m_1|m_2) \rangle = \frac{K(m_1' m_2')}{K(m_1 m_2)} \langle (m_1'|m_2') \| T^{(-1|-1)} \| (m_1|m_2) \rangle \tag{4.14}$$

for type 2 star irreps, respectively. However, the Wigner–Eckart theorem cannot be applied to $\gamma(G)$ ($\gamma(G_+)$) for type 1 (type 2) star irreps because they transform as u(1/1) tensors of different type; the decomposition of a type 1 star irrep with a type 2 star irrep is not completely reducible.

This procedure can be extended so as to be applied to the general osp$(2m/2n)$ in u(m/n) basis. But the calculation will be more complicated.

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